Received: *January 5, 2000*, Accepted: *February 7, 2000*

Mirror symmetry for $\mathcal{N}=1$ QED in three dimensions

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Abstract: We construct three-dimensional $\mathcal{N}=1$ QED with N_f flavors using branes of type-IIB string theory. This theory has a mirror, which can be realized using the S-dual brane configuration. As in examples with more supersymmetry, the Higgs branch of the original theory gets mapped into the Coulomb branch of the mirror. We use parity invariance to argue that these branches cannot be lifted by quantum corrections.

Keywords: Duality in Gauge Field Theories, Brane Dynamics in Gauge Theories, Supersymmetry and Duality.

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Contents

1. Introduction

Mirror symmetry provides a relation between the infrared descriptions of different supersymmetric gauge theories in three dimensions. The first example of this kind was discovered in ref. [1]. $\mathcal{N}=4$ supersymmetric QED with N_f flavors has a dual description in terms of another three dimensional gauge theory. The Coulomb branch of one theory is mapped into the Higgs branch of the other and vice versa, which implies that the IR descriptions of these theories agree after suitable identifications. The exact change of variables relating these theories in the infrared is not known, but recently some progress in that direction has been made [2].

While mirror symmetry was first discovered in a purely field theoretic context, it proved very useful to rederive and generalize the results of ref. [1] using brane configurations in type-IIB string theory [3] (see also [4, 5] for different approaches and [6] for a list of such brane configurations). The three-dimensional field theory is realized on D3-branes that terminate on NS5-branes and intersect D5-branes. The S-dual of this configuration gives rise to the mirror theory.

Mirror symmetry can be generalized to include theories with $\mathcal{N}=2$ supersymmetry in three dimensions [7, 8, 9]. These theories can also be realized on D3-branes in the presence of D5- and NS5-branes, and S-duality relates brane constructions that give rise to theories that are mirror pairs. A different kind of IR duality between such $\mathcal{N}=2$ theories was discussed in [10, 11].

$\mathcal{N}=4,2$ theories in three dimensions can be obtained by dimensional reduction of four-dimensional $\mathcal{N}=2,1$ theories. Just like in the four-dimensional case, there are strong constraints from R-symmetries, holomorphy, and non-renormalization theorems that simplify the analysis of the vacuum structure. For theories with eight supercharges ($\mathcal{N}=4$ in three dimensions) it is possible to show that the exact metric on the Coulomb branch of the original theory agrees with the metric on the Higgs branch of the mirror. In the $\mathcal{N}=2$ case (four supercharges) one can show that the complex structure of corresponding branches agrees.

In three dimensions one can also study $\mathcal{N}=1$ theories with two real supercharges. These theories cannot be obtained by dimensional reduction of higher dimensional supersymmetric field theories. The $\mathcal{N}=1$ superconformal algebra does not contain any R-symmetries, there are no constraints from holomorphy, and there are no know non-renormalization theorems. Nonetheless one can learn something about the infrared behavior of some of these theories (see e.g. [12, 13, 14]).

In this note we analyze three-dimensional $\mathcal{N}=1$ QED with N_f flavors and its mirror using 3-branes and 5-branes of type-IIB string theory. This theory does not have a Coulomb branch, since the $\mathcal{N}=1$ vector multiplet does not contain any scalars. We argue that parity invariance prevents the Higgs branch from being lifted by quantum corrections.[1] The S-dual brane configuration gives rise to the mirror theory with a Coulomb branch that also cannot be lifted by quantum corrections. We suggest that these two theories are equivalent in the infrared. Unfortunately it is hard to assemble additional evidence for this duality, since the metric on Higgs branch receives perturbative corrections at all orders and it does not have a complex structure we could compare to that of the mirror branch. However, it seems highly plausible that a pair of S-dual brane configurations should give rise to the same infrared physics on the 3-branes.

2. $\mathcal{N}=1$ mirror symmetry

A brane configuration consisting of NS-branes in (012345), NS′-branes in (012389), N_f D5-branes in (012348), and D3-branes in (0126) preserves two supercharges and gives rise to a $\mathcal{N}=1$ theory in three dimensions. The brane configuration is shown in figure 1. It is straightforward to determine the matter content of this theory. Each D5-brane provides one $\mathcal{N}=4$ hypermultiplet consisting of two complex scalars, Q, $\tilde{Q}$, and fermions. We can combine these fields into a complex SU(2) doublet $\mathcal{Q}=(Q,\tilde{Q}^\dagger)$. This SU(2) is a remnant of the $\mathrm{SU}(2)_R$ symmetry of the $\mathcal{N}=4$ theory, but here it acts as an ordinary global symmetry of the theory. Without the D5-brane, this configuration is $\mathcal{N}=2$ supersymmetric, so the theory contains an $\mathcal{N}=2$ vector multiplet which contains an $\mathcal{N}=1$ vector superfield and a real scalar superfield. The expectation value of the real scalar, ϕ, corresponds to the X_3 position of the three brane. In an $\mathcal{N}=2$ theory the scalar in the gauge multiplet couples to the matter fields via terms of the form $|\phi Q|^2+|\phi\tilde{Q}|^2$ in the potential. In our configuration moving the 3-branes in the X_3 direction does not give a mass to the fundamentals, so our field theory differs from the $\mathcal{N}=2$ case by turning off this coupling. Since we are considering a U(1) theory, this scalar is uncharged under the gauge group and we expect it to decouple from the low energy dynamics of the gauge theory.

[1]This argument is due to M. Berkooz, A. Kapustin, and M. Strassler [14].

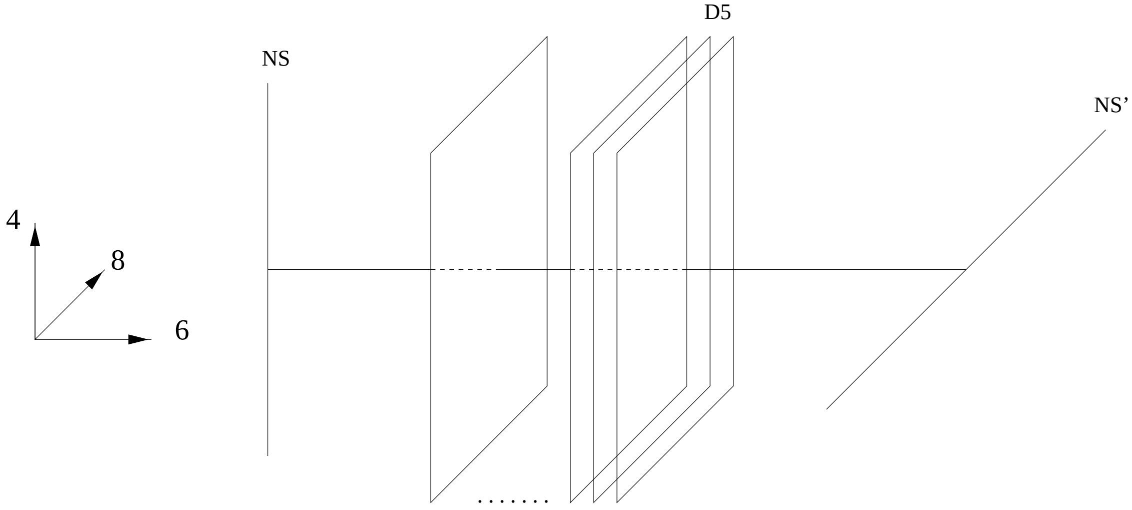

Figure 1: IIB brane configuration for $\mathcal{N} = 1$ QED. The horizontal line is the D3-brane and the other branes are labeled in the figure.

The brane configuration has no rotational symmetries, reflecting the fact that the $\mathcal{N} = 1$ superconformal algebra in three dimensions does not include an R-symmetry. The Higgs branch of the theory can be seen geometrically by breaking the 3-branes on the D5-branes and separating the pieces. For example, if there is only one D5-brane, we can separate the two halfs of the 3-brane in the $X_{3,4,8}$ directions. The X_3 position of the center of mass corresponds to the expectation value of the decoupled scalar. We expect a three-dimensional Higgs branch in this case, because it can be parametrized by expectation value of the fundamentals modulo gauge transformations, which eliminate one of the four real parameters in $\mathcal{Q}$.

The theory has three real mass parameters for each flavor. In the brane picture they correspond to motions of the D5-branes in $X_{5,7,9}$. These mass terms appear in the lagrangean in the same way as in a $\mathcal{N} = 2$ theory, i.e. they provide a complex and a real mass. Unlike in the $\mathcal{N} = 2$ case, the real mass cannot be absorbed into a shift of the real scalar in the gauge multiplet, because there is no coupling of this field to the fundamentals in the theory we discuss here.

This exhausts the list of possible brane motions. In particular there is no brane motion that corresponds to turning on FI parameters in the field theory. This is consistent with our expectations, because the $\mathcal{N} = 1$ vector multiplet does not contain any scalars.

To summarize, the classical 3-brane theory is a $\mathcal{N} = 1$ supersymmetric U(1) gauge theory with N_f complex fundamentals, $Q_i, \tilde{Q}^i$, a decoupled real scalar, ϕ, and no potential. The classical Higgs branch has $4N_f - 1$ real dimensions. There are no non-renormalization theorems that protect the Higgs branch in theories with two supercharges, so the classical Higgs branch could be lifted by quantum corrections. However, discrete symmetries of the field theory provide some constraints [12, 14].

Using the notation in ref. [12], we can define a parity transformation, P, that acts on the coordinates as $P(X_0, X_1, X_2) \to (X_0, -X_1, X_2)$ and on the spinors as $P\chi \to \pm\sigma_3\chi$. The lagrangean of the theory we described above is invariant under this transformation if all fields are parity even. There is no parity anomaly, since the number of charged fields is even. This ensures that parity is preserved in the full quantum theory. The superpotential in a parity invariant theory must be parity odd, because the the measure $d\theta^2$ is odd [12]. Since the theory we consider here does not contain any parity odd fields, we conclude that no superpotential can be generated. This in turn implies that the Higgs branch cannot be lifted by quantum corrections. Of course this does not mean that there are no quantum corrections to the metric on the Higgs branch.

In order to find a candidate mirror theory of three-dimensional QED with N_f flavors, we perform an S-duality on the brane configuration above. This turns the NS-branes into D5-branes and vice versa. The field theory on the 3-brane is now a $\mathrm{U}(1)^{N_f-1}$ gauge theory. Each U(1) factor contains an $\mathcal{N}=4$ vector multiplet, which decomposes into three real scalar superfields and one vector superfield in $\mathcal{N}=1$ language. There is a $\mathcal{N}=4$ hypermultiplet in the bifundamental representation at every intersection of the 3-brane with an NS brane. These fields couple in the $\mathcal{N}=4$ supersymmetric way to the vector multiplets except at the first and last NS brane. We discuss the $N_f = 1, 2$ cases is some detail. The generalization to higher N_f is straightforward.

For $N_f = 2$ the mirror is a U(1) gauge theory with two $\mathcal{N}=4$ hypermultiplets q_L, $\tilde{q}_L$ and q_R, $\tilde{q}_R$ from strings stretched across the left and right NS-branes respectively. The theory also contains two uncharged complex scalars, $M_{L,R}$. We identify their expectation values with the position of the left and right sections of the 3-brane in $X_3 + iX_4$ and $X_3 + iX_8$ relative to the middle section.

Expectation values for the three scalars in the $\mathcal{N}=4$ gauge multiplet correspond the the position of the middle section of the 3-brane in $X_{3,4,8}$. Giving an expectation value to either the neutral scalar fields or the scalars in the gauge multiplet makes the hypermultiplets massive. This implies a superpotential of the form

$$\begin{aligned} W = {} & (M_L + \phi_3 + i\phi_4) q_L \tilde{q}_L + \phi_8 (q_L^\dagger q_L - \tilde{q}_L \tilde{q}_L^\dagger) + \\ & + (M_R + \phi_3 + i\phi_8) q_R \tilde{q}_R + \phi_4 (q_R^\dagger q_R - \tilde{q}_R \tilde{q}_R^\dagger)\,, \end{aligned} \tag{2.1}$$

where the subscripts on the ϕ_i denote the directions they correspond to in the brane construction. ϕ_3 can be absorbed into a redefinition of $M_{L,R}$, so it is again a free decoupled field. The Coulomb branch of this theory can be parametrized by $M_{L,R}$, $\phi_{4,8}$ and the dual photon, which provides seven real parameters in agreement with the count in the original theory. To ensure parity invariance of this theory, we take the fundamentals to be parity even and the other fields to be parity odd.

The superpotential can be recast in a manifestly SU(2) invariant way. The hypermultiplets combine into SU(2) doublets $\mathcal{P}_{L,R} = (q_{L,R}, \tilde{q}^\dagger_{L,R})$, and the components

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of $M_{L,R}$ and $\phi_{4,8}$ can be rearranged into two triplets $\Phi^i_{L,R}$. In terms of these fields the superpotential reads

$$W = \Phi^i_L \mathcal{P}^\dagger_L \sigma_i \mathcal{P}_L + \Phi^i_R \mathcal{P}^\dagger_R \sigma_i \mathcal{P}_R \,. \tag{2.2}$$

The Coulomb branch can be parametrized by the expectation values of $\Phi_{L,R}$. Since the fundamentals are massive on this branch, any dynamically generated superpotential can be expressed in terms of $\Phi_{L,R}$. However, there are no parity odd and SU(2) invariant combinations of these fields. Thus we conclude that the Coulomb branch cannot be lifted by quantum corrections and corresponds to the Higgs branch of the original theory.

Moving the NS branes in $X_{5,7,9}$ corresponds to adding terms of the form

$$W_\xi = \Phi_L \cdot \xi_L + \Phi_R \cdot \xi_R \tag{2.3}$$

to the superpotential. The $\xi_{L,R}$ are similar to FI parameters in the $\mathcal{N} = 4$ theory, because they force the matter fields to develope an expectation value. However, since the $\mathcal{N} = 1$ vector multiplet does not contain any scalars, they are not FI parameters in the original sense. Nonetheless we will abuse notation and refer to the fields $\xi_{L,R}$ as FI parameters.

The FI parameters correspond to the three real mass terms in the original theory. As a consistency check on our superpotential we turn on ξ_L while keeping $\xi_R = 0$. This breaks the U(1) gauge symmetry and the resulting theory is a WZ model with a real triplet Φ_R, a complex doublet $\mathcal{P}_R$, and a superpotential

$$W = \Phi^i_R \mathcal{P}^\dagger_R \sigma_i \mathcal{P}_R \,. \tag{2.4}$$

This agrees with the superpotential we read off from the S-dual of the $N_f = 1$ brane configuration,

$$W = M_L q\tilde{q} + \mathrm{Im}[M_R](qq^\dagger - \tilde{q}^\dagger \tilde{q}) \,, \tag{2.5}$$

up to field redefinitions.

We can readily construct non-abelian theories using our brane configuration. If we put N_c 3-branes between the NS-branes, we get a SU(N_c) gauge theory in three dimensions. However, the analysis of the field theory in this case is much more involved. The scalar in the $\mathcal{N} = 2$ vector multiplet no longer decouples, since it is now charged under the gauge group. Since it does not couple to the fundamentals, they do not become massive if we give an expectation value to this field. This implies that we can move onto the Higgs branch (parametrized by the expectation value of the fundamentals) at any point on the Coulomb branch (parametrized by the expectation value of the adjoint scalar). Thus classically we find an interacting conformal field theory at every point of the moduli space, which is very difficult to analyze. We will not discuss the non-abelian case any further in this paper.

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3. Discussion

In this note we discussed a type-IIB brane configuration that gives rise to $\mathcal{N} = 1$ QED with N_f flavors in three dimensions. We argued that the Higgs branch of this theory cannot be lifted by quantum corrections, using the parity invariance of the theory. Since there are no non-renormalization theorems for theories with two supercharges, the metric on the Higgs branch receives quantum corrections but our argument indicates that this does not change its dimension.

For $\mathcal{N} = 2, 4$ theories, S-dual pairs of brane configurations give rise to three dimensional theories that are mirror pairs. We suggest that this is true also for the $\mathcal{N} = 1$ theory we consider here. To support this claim we showed that the S-dual brane configuration gives rise to another $\mathcal{N} = 1$ theory that has a Coulomb branch of the same dimension as the Higgs branch of original theory. Parity invariance ensures that neither of these branches can be lifted by quantum corrections. The mass parameters of the original theory are mapped into FI parameters of the mirror. To the best of our knowledge, this is the first proposed duality between theories with two supercharges.

Unfortunately the low degree of supersymmetry makes it hard to check our proposal any further. The metric on the moduli spaces on both sides have quantum corrections and to establish their equivalence, these would have to be computed. Since these branches are real manifolds, there are no complex structures we can compare. On the other hand, some support for our claim comes from the fact that the version of three dimensional QED we discuss here is very similar to the theory one obtains by giving a mass to all scalars in the vector multiplet of $\mathcal{N} = 4$ QED. The only difference between this theory and the theory we discussed here is the presence of the decoupled scalar in our case. Since it does not participate in the dynamics we expect these two theories to be closely related. In fact, adding a term $W = m^i \phi_i$, where m^i are uncharged scalars and ϕ_i are the three scalars in the $\mathcal{N} = 4$ vector multiplet to the superpotential of $\mathcal{N} = 4$ QED and perturbing the mirror by the corresponding FI terms, generates the mirror pairs we discussed above [14]. While this lends some support to our claims, it is not a very strong argument. Theories with two supercharges can undergo phase transitions as we vary the mass of the three scalars in the $\mathcal{N} = 4$ vector multiplet, so it is not clear that perturbing a pair of mirror theories with $\mathcal{N} = 4$ supersymmetry causes them to flow to mirror pairs of $\mathcal{N} = 1$ theories. The analysis in this paper suggests that this does happen in the specific case we study here.

Acknowledgments

It is a pleasure to thank Anton Kapustin for several very helpful discussions. This work was supported in part by DOE grants #DF-FC02-94ER40818 and #DE-FC-02-91ER40671.

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